Natural Strain

Logarithmic strain is the preferred measure of strain used by materials scientists, who typically refer to it as the "true strain." It was Nadai (ref. 1) who gave it the name "natural strain," which seems more appropriate. This strain measure was proposed by Ludwik (ref. 2) for the one-dimensional extension of a rod with length *l*. It was defined via the integral of *dl/l* to which Ludwik gave the name "effective specific strain." Today, it is after Hencky (ref. 3), who extended Ludwik's measure to three-dimensional analysis by defining logarithmic strains for the three principal directions.

In their classic treatise in 1960, Truesdell and Toupin (ref. 4) pointed out that all applications of Hencky's logarithmic strain measure had had difficulties because it was complex to evaluate. As a consequence, applications were (up to that point in time) limited primarily to studies wherein the principal axes of strain did not rotate in the body of the structure. With computers now being readily available, this consideration (which was valid in 1960) is no longer a constraint.

In their treatise, Truesdell and Toupin went on to say, "Such simplicity for certain problems, as may result from a particular strain measure, is bought at the cost of complexity for other problems. In a Euclidean space, distances are measured by a quadratic form, and an attempt to elude this fact is unlikely to succeed." They advocate using the "topological," quadratic strain fields of Almansi (ref. 5) or Green (ref. 6) instead of the "physical," logarithmic strain field of Hencky (ref. 3).

For investigations at the NASA Lewis Research Center, this researcher based his definition for natural strain on the Riemannian, body-metric, tensor field of Lodge (ref. 7). There was no 'eluding' this fact. The outcome was an intuitive measure for strain.

A thorough and consistent development of the strain and strain-rate measures affiliated with Hencky was documented (ref. 8), and natural measures for strain and strain-rate were expressed in terms of the fundamental body-metric tensors of Lodge. These strain and strain-rate measures, which are mixed tensor fields, were mapped from the body to space[1] in both the Eulerian and Lagrangian configurations and were then transformed from general to Cartesian fields. Then, they were compared with the various strain and strain-rate measures found in the literature. A simple Cartesian description for the Hencky strain-rate in the Lagrangian state was obtained, but unfortunately, this Cartesian result cannot be integrated (a byproduct of nonunique mappings from general to Cartesian space). Nevertheless, this solution does point the way to obtaining other integrable solutions appropriate for using the Hencky strain to construct constitutive equations.

This investigator believes that physical, rather than topological, measures of strain, although more complex in evaluation, will ultimately lead to much simpler constitutive equations for describing material behavior, especially under the conditions of large deformations that are often present during material processing. Simpler constitutive equations mean quicker characterization times, ultimately leading to faster turnaround times between the process design and final production.

[1]This is a one-to-one mapping (transformation law) between tensor fields defined on a body manifold to tensor fields defined on the spatial manifold.

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